The Mil
and the Mil Relation Formula

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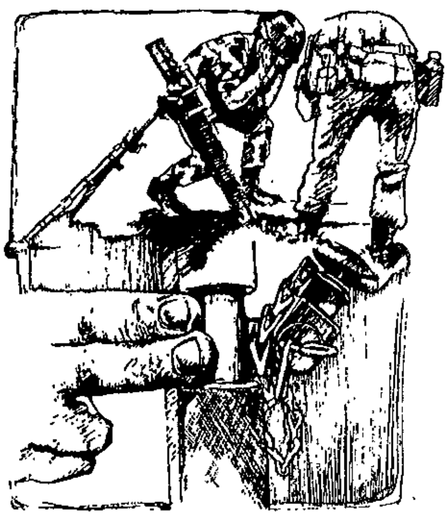
Most infantrymen are familiar with the fact that mortarmen and artillerymen use the mil (1/6,400 of a circle, or about 1/18 of a degree) instead of the more familiar degree (1/360 of a circle) as their basic unit of angular measurement. We have all been taught the mil relation or WORM (W = RM) formula, which is most commonly used for finding deviation shifts when adjusting indirect fire. But few infantrymen (including mortarmen) know where the mil and the mil relation formula came from, or more important, how accurate the formula is.

The mil was developed by the French Army in the 1890s and was originally called the *millieme* (French for "thousandth"). The credit for the invention goes to a Captain (later Major General) Estienne, who designed a new sight that was graduated in 6,400 mils and adopted in 1900 for the famous 1897 model 75mm gun. The 75 was the first field piece with an effective hydro-pneumatic recoil system, allowing it to be fired without being relaid after each round. It became the standard American direct support artillery weapon in World War I and stayed in our service through the following two decades.

The Germans copied the French 6,400-mil division of the circle before World War I, while the Russians adopted the 6,000-mil circle and have stayed with it. The first American cannon to have its deflection scale graduated in mils was the 1902 model three-inch field gun. This weapon was our first modern field piece, but its hydraulic and spring recoil system was not as good as that of the French 75.

The mil relation formula itself cannot be said to have had a sole inventor. All artillery officers in the late 19th century had to be proficient at trigonometry because of the equipment of the period and the lack of accurate maps; the ability to develop equations such as the mil relation formula on an "as needed" basis was part of their stock-in-trade. Magazine articles and



manuals of the period describe the mil as one-thousandth of the range. Thus, a mortarman firing 4 mils to the right of a target at a range of 6,000 meters would have been 24 meters off. Such relationships could have been worked out without the mil, of course, but the mil made it far easier. Consequently, official credit must again be given to Captain Estienne.

It would be nice to get an American into the picture, though, and there was such an American, Second Lieutenant (later Brigadier General) Alston Hamilton of North Carolina. Hamilton was involved in the development of a method of indirect fire in 1897, but it required a complicated instrument that was in short supply. He therefore worked out a simpler method and equipped his battery accordingly for the Spanish-American War in the following year. (American field artillery used only direct fire in those days, though, and indirect fire proved unnecessary in the Cuban campaign.) In 1899 Hamilton described his technique in an article in the *Journal of the United States Artillery* entitled "A Simple Method of Laying Guns for Indirect Fire for the 3.2 B.L. Field Rifle." (The articles of this publication, beginning in 1890, reveal that Hamilton's was the first to show the mil relation formula.)

The 3.2-inch gun that Hamilton was dealing with had its deflection scale graduated in "points," each equal to 1/6 of a degree (about 3 mils). In his article Hamilton considered the problem of concentrating the fire of his guns on a single point (a converged sheaf); he then had to determine how many points to shift each gun with respect to the adjusting gun. A mortar fire direction center today would solve this by using the mil relation formula M = W/R or by looking up the answer in the 100/R column of the firing tables. Hamilton used simple trigonometry to work out a variant of the mil relation formula for points: M = W/(3R) or W = 3RM (where the angle M is measured in points).

There are 6 points in one degree and therefore 2,160 points in a full circle. If we divide the points into thirds, we see that Hamilton's formula works out to W = RM for a circle divided into 6,480 equal parts, virtually the same as the present W = RM for a 6,400-mil circle.

It turns out that the introduction of the mil coincided with the arrival of modern artillery and the beginning of the changeover from direct to indirect fire. Modern mortars were developed a decade and a half later (during World War I), and so infantrymen found themselves having to learn about mils. World War I also saw the widespread use of telephones at the tactical level for forward observation, and the tactical use of radios soon followed. The development of extensive electronic communications meant that infantrymen were likely to find themselves adjusting artillery fire, which again required a knowledge of the mil and an understanding of its versatility.

With some simple logic and mathematics, we, too, can come to a better understanding of the whole matter, which is something that we now only memorize. This understanding will make the memorization easier and will let us see just how useful the mil relation formula really is.

First, the world "mil," as we have seen, means one-thousandth (the U.S. dollar, for example, is divided into 100 cents or 1,000 mils, and wire is measured in mils, each equal to 1/1,000 of an inch). But what is the Army's mil one-thousandth of?

To answer this question we need to consider three more questions. Why not divide the circle into 360 degrees or into 64,000 parts instead of 6,400? Why divide it into 6,400 parts instead of, perhaps, 6,283? Why not divide it into 6,000 parts as the Soviets do?

The first question is easy enough to answer. If mortarmen and artillerymen used the degree as their unit of angular measure, then to be accurate they would have to use fire commands that included decimals — for instance, "Deflection one seven nine *point* two five." Fire commands must be shouted out clearly in all kinds of weather, and having a decimal point in them would be asking for trouble. On the other hand, if we used 1/64,000 of a circle as our unit of measure, then deflection commands would be overly precise. (The field artillery does use tenths of mils in special cases, but an examination of bursting areas, deflection probable errors, and ranges of weapons from mortar and cannon firing tables show that we want a unit of measure in the general area of 1/6,400 of the circle.)

The second question (why 6,400 instead of 6,283) is also simple to answer. There may be a theoretical reason, as we will see later, why a mil equal to 1/6,283 of a circle would be better than 1/6,400, but a number like 6,283 is awkward to work with. It cannot be divided by anything, but 6,400 can be divided easily by 2 or by 5 so that sectors can be subdivided many times without using fractions (6,400 ÷ 2 = 3,200 ÷ 2 = 1,600 ÷ 2 = 800 ÷ 2 = 400 ÷ 2 = 200 ÷ 2 = 100 ÷ 2 = 50 ÷ 2 = 25 ÷ 5 = 5). The Soviet choice of 6,000 has the added advantage of being divisible by 3 as well as by 2 and 5 (6,000 ÷ 2 = 3,000 ÷ 2 = 1,500 ÷ 2 = 750 ÷ 2 = 375 ÷ 5 = 75 ÷ 5 = 15 ÷ 3 = 5). In short, it is easy to do mental arithmetic with either 6,400 or 6,000.

ACCURACY

The last question (should we use 6,000 instead of 6,400) gets to the heart of the mil relation formula: W = RM, where W = width in meters, R = range in thousands of meters, and M = angle in mils. The formula is based on the assumption that a one-mil arc subtends a distance of one meter at a range of 1,000 meters, or that one mil subtends a distance equal to 1/1,000 of the radius of a circle drawn with the observer at the center, and a radius equal to the observer-target distance. It is easy to check the validity of this assumption by using the formula for the circumference of a circle: C = 2 (Pi)r, where Pi = 3.1416 and r = radius of the circle. A circle with a radius of 1,000 meters has a circumference of 2 x 1,000 x Pi = 2,000 x 3.1416 = 6,283 meters. One mil subtends 1/6,400 of this circumference, so one mil = 6,283 ÷ 6,400 = 0.98 meter. Therefore, the assumption and the mil relation formula are 98 percent accurate for a one-mil angle. The formula slowly gets less accurate as the mil angle increases, but it is still 98 percent correct for a 100-mil angle. Accuracy then falls off more rapidly, but even for an angle of 600 mils (the maximum for which the formula is used), it is between 90 and 92 percent accurate. (The calculations for 100 and 600 mils require elementary trigonometry, and the results vary slightly depending on whether one is adjusting a burst onto a target or shifting from a registration point to a new target.)

In other words, dividing the circle into 6,400 parts means that each part will be almost 1/1,000 of the radius of the circle. The formula would be more exact, of course, if the circle were divided into 6,283 parts, but the resulting arithmetic would be too messy. Since 6,400 is slightly closer to 6,283 than is 6,000, the U.S. mil relation formula is slightly more accurate than the Soviet version (which is 95 to 96 percent accurate for angles between 1 and 100 mils).

The fact that the mil relation formula is about 98 percent accurate in most situations is worth knowing. Some infantrymen have the bad habit of assuming that their eyeball estimates are better than the formula when adjusting indirect fire. They invariably underestimate deviation errors. For instance, they call for a 50-meter shift when the formula specifies 120 meters. This wastes time that may not be available on the modern battlefield. Yet, they usually know the range fairly accurately from the map (or from flash-to-bang time) and certainly should be able to measure the mil angle pretty well with their binoculars or their fingers. All they have to remember is that the mil relation formula is a pretty good one, and that only the enemy benefits from the assumption that calibrated eyeballs are better.

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